Problem Set EARN – Economic growth models

1) Consider the aggregate Cobb-Douglas production function

$$GDP = A \cdot K^{\alpha} \cdot L^{\beta}$$

   i) Consider that $\alpha + \beta = 1$. How much will GDP increase if both the capital and labour factors of production are tripled? Show it mathematically. What is the name of this property of the Cobb-Douglas production function?

   ii) If $\alpha + \beta > 1$, and you triple both factors of production, do you expect GDP to be higher than in i)? Why?

   iii) Consider that $\alpha = 0.3$ and $\beta = 0.7$. How much do you expect GDP to increase, in percentage, if you increase labour by 1%, keeping all else in the production function constant? What is the name given to the exponents α and β?

   iv) Consider that $\alpha = 0.3$ and $\beta = 0.7$. If the levels of production, capital, and labour are as follows:

   GDP: 109 K: 70 L: 30

   What is the value of total factor productivity A?

2) Name the two traditional factors of production in neoclassical economic growth theory, and provide a brief definition for each.

3) Consider the aggregate Cobb-Douglas production function

$$GDP = 60 \cdot K^{1/3} \cdot L^{2/3}$$

   i) Complete the following table

K	L	GDP
64	8	
128	16	
192	24	

   ii) Explain how the above results illustrate the property of constant returns to scale.

   iii) Mathematically derive the expression for the marginal product of labour MP_L.

   iv) Evaluate the expression you derived in iii) – that is, find the numerical value of MP_L, when $K = 64$ and $L = 8$.

4) Consider an economy in which the level of capital stock available for production, at the beginning of year 2007, is 1600€. During that year, the consumption of fixed capital is 50€.
   i) Throughout 2007, this economy decided to invest 80€ in new capital for production. Determine the level of capital stock available for production in 2008. What is the name of the method you've used?
   ii) What is the depreciation rate of capital stock for year 2007?
   iii) What is the capital accumulation rate between 2007 and 2008?
   iv) Assuming that this economy's GDP can be described by an aggregate production function of the Cobb-Douglas type, and knowing that capital is paid approximately 30% of total income, how much will GDP increase, in percentage, from 2007 to 2008, from the increase in capital stock between those years, keeping all else constant?

5) In a given economy, the contribution of human labour to production and economic growth is determined by measuring the number of people in the labour force.
   a) Give an example of someone who is part of the labour force, and an example of someone outside the labour force.
   b) Can you suggest a better measure to account for human labour in production? What?
   c) What could drive the labour force of a country to grow? And to decrease?

6) When measuring human labour contribution to economic production, qualitative differences can be measured by computing a human capital index

$$h = 2 \cdot \varphi(s)$$

With $\varphi(s)$ being a function of returns to additional years of schooling, given by

$$\varphi(s) = \begin{cases} 0.432 \cdot s, & if\ s \leq 5 \\ 0.432 \cdot 5 + 0.238 \cdot (s-5), & if\ s > 5 \end{cases}$$

   a) Suppose that the average years of schooling in the country, in 1960, is $s = 4$. What are the average returns φ? What is value for the human capital index in that year?
   b) If the average years of schooling increase to 5 in 1965, what are now the values for the average returns and human capital index? How much has the human capital index increased between 1960 and 1965?
   c) If the average years of schooling increase again in 1970, to 6, what are now the average returns and human capital index?
   d) Finally, assume that between 1965 and 1970, the number of workers employed in the country grew from 10 000 to 20 000. How much has the human labour used in production increased, measured without accounting for the skill level of workers? And how much has it increased now accounting for the skill level? Compare and discuss.

7) Consider an economy in which capital stock available for production in 2018 is 1500€, and human labour (in hours), for the same year, is 5000.
   i) Throughout the year 2018, investment in capital stock is 375€, while capital depreciates at a rate of $\delta = 5\%$. How much does capital stock grow, in percentage, from 2018 to 2019?
   ii) Assuming that human labour grows at a rate of 10% from 2018 to 2019, and based only on the contribution from capital and labour, determine GDP in 2019, using a Cobb-Douglas production function (i.e. $GDP = K^{\alpha} \cdot L^{1-\alpha}$, with $\alpha = 0.3$).
   iii) Using the same method in line ii), determine GDP in 2018, and compute its growth rate from 2018 to 2019.
   iv) Actually, historical GDP in this economy was, in 2018, 8000€, and in 2019, 11200. Compute the total factor productivity (A) in both years.
   v) Do a growth accounting exercise and compute the contributions of capital, labour, and total factor productivity, to economic growth between 2018 and 2019.

8) Consider the economic model, given by the following equations:

$$GDP = A \cdot K^{\alpha} \cdot L^{1-\alpha}, \qquad A = \left(\frac{EFF_t}{EFF_0}\right)^{\beta}, \quad \frac{U_{ex}}{GDP} = 1, \qquad \frac{U_{ex}}{F_{ex}} = EFF_t$$

Where U_{ex} is useful exergy consumed, F_{ex} is final exergy consumed, EFF_t is the final-to-useful exergy efficiency for period t ($t = 0$ refers to the initial year). Assume that $\alpha = 0.3$ and $\beta = 1.2$.

   a) Assuming that final-to-useful exergy efficiency increases by 1% from one year to the next, and keeping all else constant, what do you expect the increase to be in
      i. Total factor productivity A?
      ii. Gross domestic product GDP?
      iii. Useful exergy U_{ex}?
   b) Assume now the following value, for two years of this economy:

Year	K	L	EFF	A	GDP	U_{ex}	F_{ex}
2004	81	9	20%				
2005	162	18	24%				

      i. Knowing that the value for final-to-useful efficiency, at the initial year 1960, is $EFF = 10\%$, what are the normalized values for these efficiencies in 2004-2005?
      ii. What are the values for total factor productivity A and GDP in 2004-2005? What are the growth rates for these values, from 2004 to 2005?
      iii. Determine the values of U_{ex} and of F_{ex}, for 2004 and 2005. Is there a rebound effect? Explain.

9) Describe what is understood by the energy efficiency rebound effect, and distinguish between direct and indirect types of this effect, resorting to examples if needed.